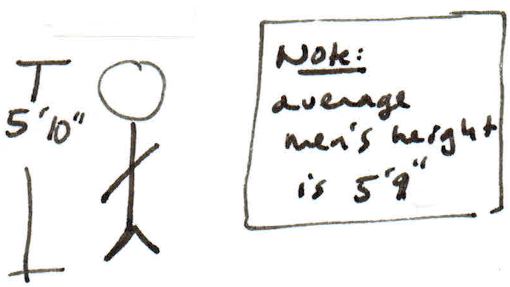
Sketching for STEM

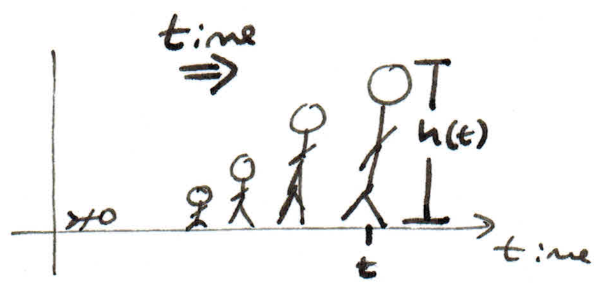
A sketch has 4 parts

1. drawing
2. dimensions & labels
3. arrows
4. notes

Static & Dynamic Sketches

Eg:





Static

Dynamic

Calculus!

To indicate motion / change

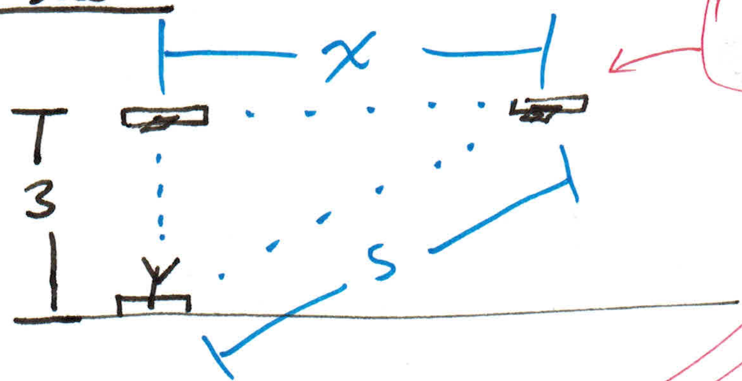
1. dimension using functions
2. draw 2nd copy & unknown time
3. use arrows

Worksheet Eg 1

A plane Flies constant alt. 3mi

2nd sketch @ indeterminate time later

(0) Sketch



$\boxed{\text{Dynamic.}}$ $\Rightarrow$ dimension w/ functions.

Notation

let s be distance from station

x be horizontal distance

know $\frac{ds}{dt}$ when $s=5$

want $\frac{dx}{dt}$ when $s=5$

(1) Relate the functions

Dynamic picture $\Rightarrow$ S & X are FUNCTIONS (dimensions)

$$3^2 + x^2 = s^2$$

(2) Relate the Rates

$$\frac{d}{dt}\left[3^2 + x^2\right] = \frac{d}{dt}\left[s^2\right]$$

$$\cancel{2}\cdot x \cdot \frac{dx}{dt} = \cancel{2}\cdot s \cdot \frac{ds}{dt}$$

(3) <u>Finish the problem</u>

<u>Want</u> $\frac{d\cancel{x}}{dt}$ when $s=5$

<u>Know</u> $\frac{ds}{dt} = 500$ when $s=5$

$$x \cdot \frac{dx}{dt} = 5 \cdot 500$$

when $s=5$ what is x?

$$3^2 + x^2 = 5^2$$

$$9 + x^2 = 25$$

$$x^2 = 16$$

$$x = 4$$

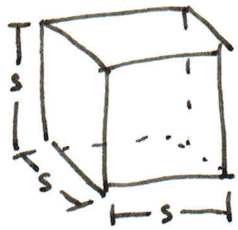
$$4 \cdot \frac{dx}{dt} = 2500$$

$$\boxed{\frac{dx}{dt} = \frac{2500}{4} = 625 \text{ mi/hr}}$$

Worksheet Eg 2

An ice cube melts.

(0) sketch



Note: surface area

6 equal sides, each of area s^2

$$A = 6s^2$$

Given: $\frac{dA}{dt} = -3 \ in^2/s$

Want: $\frac{ds}{dt}$ when $s = 1$.

(1) Relate Functions

$$A = 6s^2$$

(2) Relate Rates

$$\frac{d}{dt}[A] = \frac{d}{dt}[6s^2]$$

$$\frac{dA}{dt} = 6 \cdot 2s \cdot \frac{ds}{dt}$$

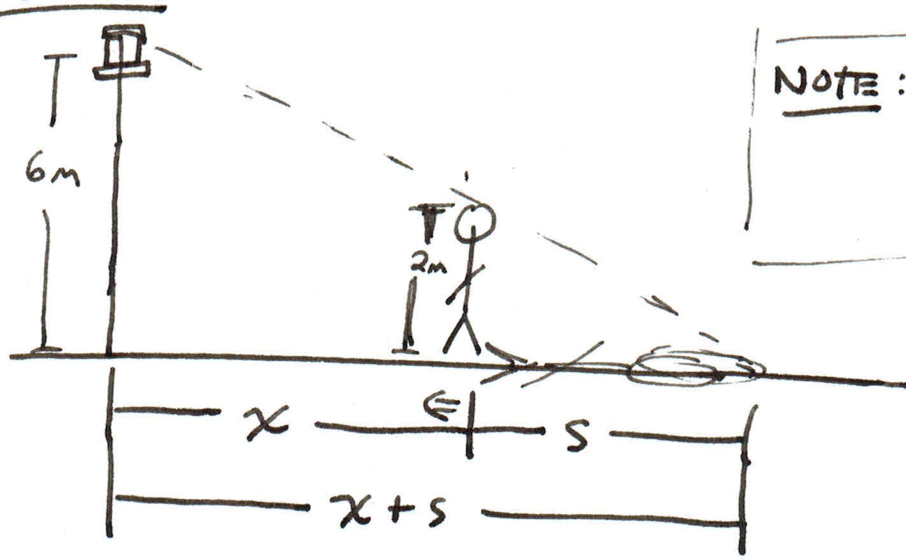
(3) answer Qn when $s = 1$

$$-3 = 6 \cdot 2 \cdot 1 \cdot \frac{ds}{dt} \Rightarrow \boxed{\frac{ds}{dt} = \frac{-3}{12} = \frac{-1}{4} \ in/s}$$

Worksheet Eg 3

A streetlight is mounted above a 6 m pole...

(0) Sketch



NOTE: by similar △'s

$$\frac{s}{2} = \frac{x+s}{6}$$

Given: $\frac{dx}{dt} = -2$

Want: $\frac{ds}{dt}$ when $x = 4$. & when $x = 1$.

(1) Relate the Fns

$$\frac{s}{2} = \frac{x+s}{6}$$

$$6s = 2(x+s)$$

$$6s = 2x + 2s$$

$$4s = 2x$$

$$2s = x$$

(2) Relate the Rates

$$\frac{d}{dt}[2s] = \frac{d}{dt}[x]$$

$$2 \cdot \frac{ds}{dt} = \frac{dx}{dt}$$

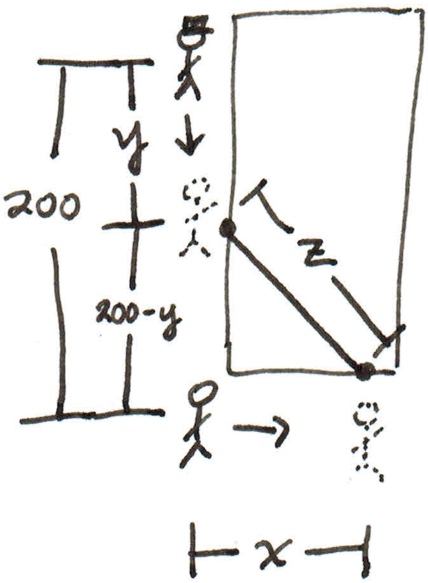
(3) Answer Qn

for every x,

$$2 \cdot \frac{ds}{dt} = -2$$

$$\boxed{\frac{ds}{dt} = -1}$$

Eg #4 A police officer is walking down a street



where y = dist officer travels

x = dist felon travels

z = dist between.

Dynamic!

→ 2^{nd} pic @ indeterminate time

→ dimension using functions.

GIVEN: $\frac{dy}{dt}$ & $\frac{dx}{dt}$

WANT: $\frac{dz}{dt}$

(1) Relate the Functions

$$(200-y)^2 + x^2 = z^2$$

(2) Relate the Rates

$$\frac{d}{dt}\left[(200-y)^2\right] + \frac{d}{dt}\left[x^2\right] = \frac{d}{dt}\left[z^2\right]$$

$$= 2\cdot(200-y)\cdot\frac{d}{dt}[200-y] + 2\cdot x\cdot\frac{dx}{dt} = 2\cdot z\cdot\frac{dz}{dt}$$

$$2\cdot(200-y)\cdot\left(-\frac{dy}{dt}\right) + 2x\frac{dx}{dt} = 2z\cdot\frac{dz}{dt}.$$

(3) Answer the Qn.

NOTE: velocity constant

$\Rightarrow \quad x = 9t$

& $\quad y = 12t$

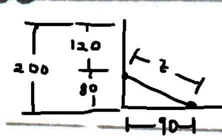
So

when $t = 10\,s$

$x = 90\ ft$ & $\dfrac{dx}{dt} = 9$

$y = 120\ ft$ $\quad \dfrac{dy}{dt} = 12$

and $\quad z^2 = 90^2 + (200-120)^2$



$\Rightarrow \quad z = \sqrt{90^2 + 80^2}$

$$2(200-120)\cdot(-12) + 2\cdot(90)\cdot 9 = 2\sqrt{90^2+80^2}\,\frac{dz}{dt}$$

solving for $\dfrac{dz}{dt}$

...

$$\boxed{\frac{dz}{dt} \approx -1.24\ldots\ ft/s}$$

10 sec into the chase, distance from officer to felon decreasing at $\approx 1\ m/s$,

use similar triangles to set up

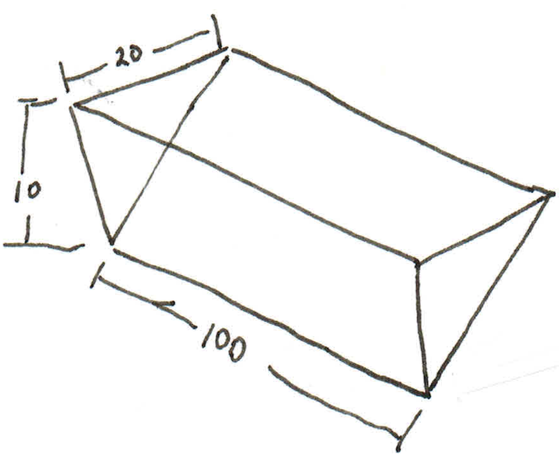
Eg: Suppose there's a 100 m long water trough w/ cross section ∇ (a rectangular prism)

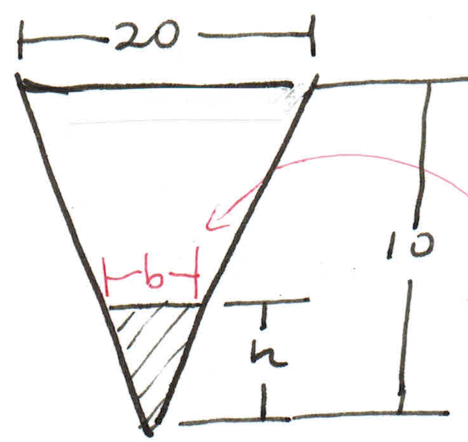
20 m accross top, 10 m tall.

Filling at constant rate of 400 m^3/hr. Given.

How fast is height changing when water is halfway up side? Want

① Sketch to relate the functions





Static picture **NOT** enough

NEED dynamic picture

NOTE volume of water $= \begin{pmatrix} \text{end} \\ \text{area} \end{pmatrix} \cdot \begin{pmatrix} \text{length} \end{pmatrix}$

end area $= \frac{1}{2} \cdot h \cdot b$

By similar $\triangle$'s $\quad \frac{b}{h} = \frac{20}{10}$

2 unknowns (h & b)
$\Rightarrow$ need 2 eqns.

(1) Relate the Functions $\leftarrow \boxed{h \,\&\, V}$

$$V = \begin{pmatrix} \text{end} \\ \text{area} \end{pmatrix} \cdot \begin{pmatrix} \text{length} \end{pmatrix}$$

$$\boxed{\text{end area} = \tfrac{1}{2} \cdot b \cdot h}$$

$$V = \frac{1}{2} \cdot b \cdot h \cdot 100$$

$$\frac{b}{h} = \frac{20}{10}$$

$$10\,b = 20\,h$$

$$b = 2h$$

$$V = \frac{1}{2} \cdot 2h \cdot h \cdot 100 = 100\,h^2$$

$$V = 100\,\boxed{h}^2$$

implicit fn

(2) Relate the Rates

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[100\,\boxed{h}^2\right]$$

$$\frac{dV}{dt} = 100 \cdot 2h \cdot \frac{dh}{dt}$$

(3) Answer the Question

water is half way up when

$$h = \frac{10}{2} = 5$$

$\boxed{\text{So}}$

$$400 = 100 \cdot 2.5 \cdot \frac{dh}{dt}$$

$$\frac{400}{1000} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{10} = 0.4 \ m/hr$$

Worksheet Problem #6

Suppose the water trough leaks.

Then

$$\frac{dV}{dt} = \begin{pmatrix}\text{rate of} \\ \text{water in}\end{pmatrix} - \begin{pmatrix}\text{rate of} \\ \text{water out}\end{pmatrix}$$

$$= 400 - 100$$

$$\frac{dV}{dt} = 300 \ \frac{cm^3}{s}$$

the rest of the problem is the same as it was in #5

Related Rates Worksheet | Math 1131 | Class #21

Name: ______________________ Section: ______________________

Principles of Sketching[1]

There are four basic elements of a sketch

1. **The Drawing:** Sketch the physical objects being described. Try to match the scale and relations between things.
2. **Annotations:** Add names, labels, and explanatory notes.
   - Label quantites that change over time with *letters.* If a quantity (length, angle, etc) does *not* change over time, you can label the drawing with its value.
   - You might also want to add additional lines to create a shape like a triangle, which can be used along with trigonometry or the Pythagorean theorem.
3. **Arrows:** Draw arrows to indicate motion. Once drawn, these arrows can often help you find out where to fill in the missing lines to create a triangle.
4. **Notes:** Next to your drawing, write down any formulas that may be useful for relating the relevant quantities. Common examples are area, volume, trig, and distance formulas. You may also use facts about similar triangles.

Useful Formulas

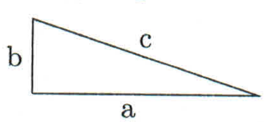
1. Area

shape	formula
circle	πr^2
triangle	$\frac{1}{2}bh$
rectangle	bh
sector of a circle	$\frac{1}{2}\pi r^2 \cdot \theta$

2. Volume

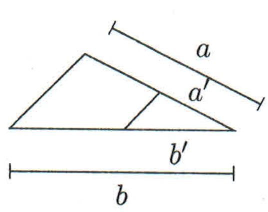
shape	formula
sphere	$\frac{4}{3}\pi r^3$
cylinder	$\pi r^2 h$
cone	$\frac{1}{3}\pi r^2 h$
triangular prism	$\frac{1}{2}bhl$
rectangular prism	bhl

3. If a solid has a constant cross-section, its volume equals its surface area times its length.
4. The distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
5. The Pythagorean theorem for right triangles.



$$a^2 + b^2 = c^2$$

6. In similar triangles, corresponding sides have the same proportion.



$$\frac{a'}{a} = \frac{b'}{b} \quad \text{and therefore} \quad a' \cdot b = b' \cdot a$$

[1]Adapted from §3.4 of *Sketching User Experiences: The Workbook*, by Greenberg et.al.

Name: ____________________ Section: ____________________

For each problem

(a) What is given?

(b) What is unknown?

(c) Sketch a picture of the situation at some unknown time t.

(d) Write an equation that relates the quantities.

(e) Finish solving the problem

1. An airplane flies directly over a radar station, at a constant altitude of 3 mi above the ground. A little while later, the radar station measures that (a) the distance between the plane and radar station equals 5 mi and (b) that the distance between the plane and radar station is increasing at a rate of 500 mi/hr. What is the ground speed of the airplane at the time of the second measurement?

2. An ice cube melts, with its surface area decreasing at a rate of 3 in^2/s. How fast is the side length decreasing when the side length is 1 in?

3. A streetlight is mounted at the top of a 6 meter pole, and a 2 meter tall person is walking toward it at 2 meters per second. How fast is the length of their shadow changing when they are 4 meters from the streetlight? What about when they are 1 meter from the light?

4. A police officer is walking down a city street, when they spot a wanted felon standing 200 ft away at the corner of the next block. The police officer takes off after the felon at 12 ft/s, and the felon immediately cuts around the corner and runs away at 9 ft/s. What is the rate of change of the distance between the officer and the felon after 10 seconds have passed?

5. Suppose there is a 100 cm long water trough shaped as a triangular prism whose cross-section is an inverted triangle ∇ which is 20 cm across the top, and is 10 cm tall. If the tank is being being filled with water at a constant rate of 400 cm^3/s, how fast is the height changing when the water is half way up the side of the tank?

6. Suppose the water trough above leaks (100 cm long, cross section is a ∇, top $=$ 20 cm, and height $=$ 1 cm). If water is being added to the tank at a rate of 400 cm^3/s, and is leaking out of the tank at 100 cm^3/s, how fast is the height changing when the water is half way up the side of the tank?